Characterization of Cylindrical Cloak by Unequal Multi-Layer Homogeneous Isotropic Materials

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***Abstract*—The perfect cylindrical cloaks require anisotropic material parameters, which are very difficult to realize in practice. An effective approach to realize electromagnetic cloaking is that the cloak composed of homogeneous isotropic materials is concentric layered instead of using the metamaterial. For most cases, material parameters of the cloak are nonlinear, we propose a concentric layer structure with different thickness layer to realize ideal cylindrical cloak with spatially invariant axial material parameters. Numerical simulation results verify that, when the layer number is identical, the proposed cloaking exhibits better invisibility performance compared to the cloak previously publications, that is, the cloak with the equal thickness layers.**

I. Introduction

Based on coordinate transformations, cloak of invisibility was proposed by J. Pendry, which can perfectly conceal arbitrary objects from the coming detection [1].Following this approach, the experimental realization at microwave frequency was soon reported, and various cloaks and other interesting devices such as concentrator, field shifter have been proposed afterwards[2-20]. The ideal cylindrical cloak has three spatial variant material parameters, which is very difficult to realize in practice. Then cylindrical cloak designed with simplified parameters are proposed aiming to decrease the complexity in the realization of cloak, but exit finite scattering [21]. Perfect cylindrical invisibility cloak with spatially invariant axial material parameters were proposed, then difficulties in the experimental realization of the cloak were reduced, but good performance of invisibility are still keeping[22]. For most cases, electromagnetic invisible cloak requires inhomogeneous and anisotropic media, therefore it is difficult to construct by using naturally existed materials. Y. Hung proposed a cylindrical cloak structure with homogeneous materials [23]. It was realized by a concentric identical thickness layered structure consisting of alternating homogeneous isotropic materials, which can be treated using theory of effective medium as anisotropic layer.

In this paper, an ideal cylindrical cloak with the different thickness concentric layered structures was effectively constructed. It can be realized by layer structures of composite media. Numerical simulation results validate that, with the same layer number, the cloak with the different thickness layers exhibits better invisibility performance compared to the cloak previously publications, that is, the cloak with the equal thickness layers.

II. Cylindrical Cloak With Different Thickness Layer Structure of Homogeneous Materials

Material parameters for the perfect electromagnetic cylindrical invisibility cloaks are suggested by Reference [22]:

$$\varepsilon_r = \mu_r = \frac{r^2 - a^2}{r^2}, \tag{1}$$

$$\varepsilon_\theta = \mu_\theta = \frac{r^2}{r^2 - a^2}, \tag{2}$$

$$\varepsilon_z = \mu_z = \frac{b^2}{b^2 - a^2}, \tag{3}$$

Where a is the inner radius, b is outer radius, and the cloak occupies the cylindrical region ($a \prec r \prec b$).

$\varepsilon_r, \mu_r, \varepsilon_\theta, \mu_\theta$,

ε_z and μ_z represents radial, angular and z components of permittivity and Permeability respectively.

As proposed in Reference [23], the cylindrical invisible cloak with inhomogeneous and anisotropic media can be constructed through layered structures of homogeneous isotropic materials,

and the structure is shown in Fig 1.

The cloak is divided into N equal-thickness layers, and the layer is composed of two sub-layers. The each homogeneous anisotropic layer is mimicked by two sub-layers with different parameters, and the parameters of the two sub-layers were derived from the corresponding anisotropic layer using theory of effective medium.

According to effective medium theory, when the layer thickness is much less than the wavelength, we can treat the material of above structure as a single anisotropic medium with the permeability:

$$\mu_r = \frac{(1+\eta)\mu_1\mu_2}{\mu_2 + \eta\mu_1} \tag{4}$$

$$\mu_\theta = \frac{\mu_1 + \eta\mu_2}{1+\eta} \tag{5}$$

Where $\eta = d_1/d_2$ is the ratio of two sub-layer thicknesses. μ_1 and μ_2 represent the permeability of sub-layer respectively.

For most cases, material parameters of the cloak are nonlinear, so discrete parameters of the equal thickness can't well fit anisotropic parameters. Fig 2 shows the curves of μ_r and μ_θ proposed in Reference [19].

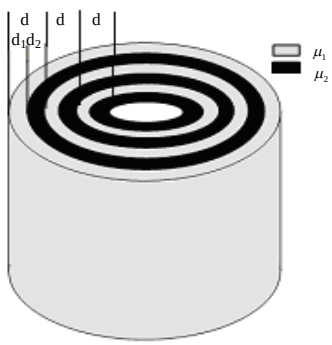
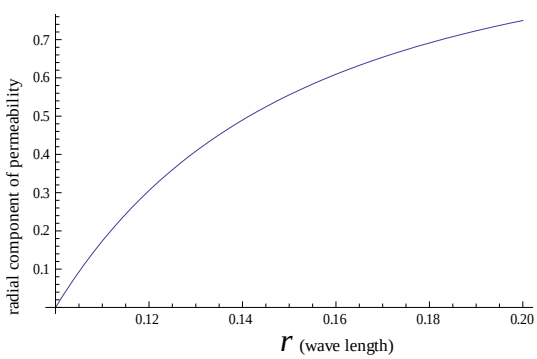
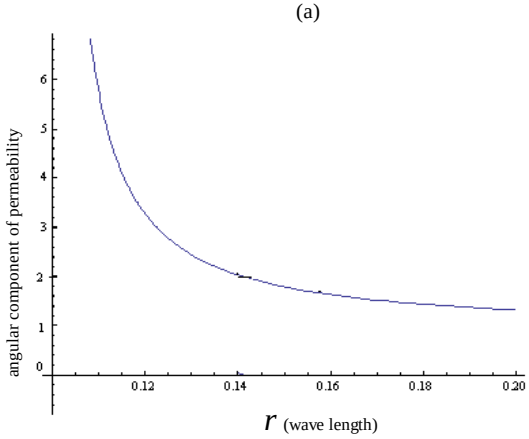


Fig 1 Concentric layer structure with equal thickness



(a)



(b)

Fig 2 Curves of μ_r and μ_θ based on coordinate transformation

It can be clearly seen that μ_r and μ_θ changes lightly near the outer region compared to the inner region. The discrete parameter derived from equal-thickness layers can't mimic the original continuous parameters well. So we can discrete the parameter according to gradient, that is, more layers are divided in the region of great gradient compared to small gradient. In this paper, an ideal cylindrical invisibility cloak with spatially invariant axial material parameters implemented by different thickness concentric layer structures was constructed. In order to contrast to the identical thickness layer structure previous publication, we divide the cloak into 10 layers. Here, we assume η=1, the cylindrical cloaking shell with inner radius a= 0.1m and outer radius b = 0.2 m, anisotropic medium could be represented approximately by ten unequal discrete layers of homogeneous anisotropic medium. According to Fig.2 and Fig.3, near the region of outer radius, the gradient is not steep, so we use one layer to substitute two layers of identical thickness. near the region of the inner radius, the gradient is steep , so we divide original one identical thickness layer into two layers.

Combining equations (1), (2), (4) and (5), the parameters of the different thickness layer structure cloak (N=8 and 10) and the parameters of the equal thickness layer structure cloak (N= 10) are shown in table 1, table2 and table 3 respectively.

To demonstrate the performance of the designed cloak, full-wave simulations with the commercial finite-element solver COMSOL MULTIPHYSICS was performed to verify the performance of the designed electromagnetic cloak. We assume TE plane wave incidence along the x direction at 2 GHz frequency (or $\lambda = 0.15$ m), the cloaking materials are assumed to be lossless and the interior region is perfect electric conductor (PEC), and perfect matched layers (PML) are used to absorb the scattered field, and boundary settings are continuity. The simulation results of the proposed cloaking structure are shown in Fig.3, Fig.4 and Fig.5.

Table 1 Permeability of eight-layer cloak with different thickness

Layer(unit: λ)	2.0-1.8	1.8-1.6	1.6-1.45	1.45-1.3
μ_1	0.4514	0.4014	0.3399	0.2832
μ_2	2.2153	2.4915	2.9422	3.5308
Layer(unit: λ)	1.3-1.2	1.2-1.1	1.1-1.05	1.05-1.0
μ_1	0.2134	0.1565	0.0874	0.0466
μ_2	4.6851	6.3889	11.4364	21.4565

Table 2 Permeability of ten-layer cloak with different thickness

Layer(unit: λ)	2.0-1.8	1.8-1.6	1.6-1.45	1.45-1.3	1.3-1.25
μ_1	0.4514	0.4014	0.3399	0.2832	0.2134
μ_2	2.2153	2.4915	2.9422	3.5308	4.6851
Layer(unit: λ)	1.25-1.2	1.2-1.15	1.15-1.1	1.1-1.05	1.05-1.0

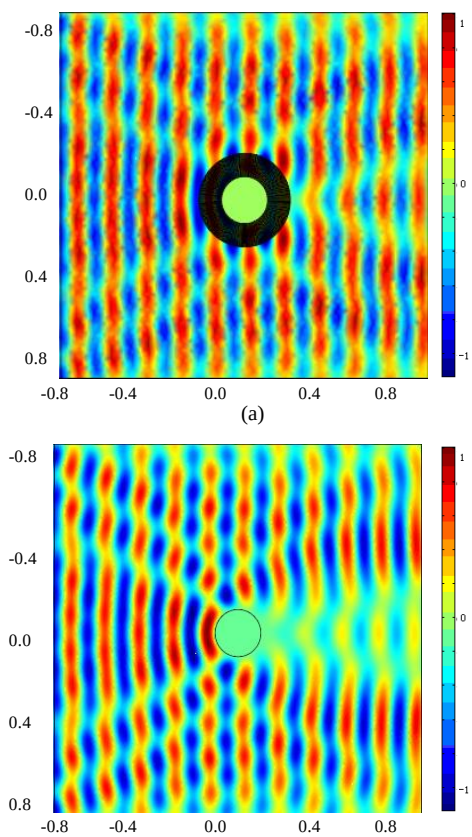
μ_1	0.1862	0.1565	0.1238	0.0874	0.0466
μ_2	5.3693	6.3889	8.0778	11.4364	21.4565

Table 3 Permeability of ten-layer cloak with equal thickness

Layer(unit: λ)	2.0-1.9	1.9-1.8	1.8-1.7	1.7-1.6	1.6-1.5
μ_1	0.4514	0.0427	0.4014	0.3723	0.3399
μ_2	2.2153	2.3387	2.4915	2.6859	2.9422
Layer(unit: λ)	1.5-1.4	1.4-1.3	1.3-1.2	1.2-1.1	1.1-1.0
μ_1	0.3303	0.2617	0.2134	0.1565	0.0874
μ_2	3.2967	3.8217	4.6851	6.3889	11.4364

As shown in Fig.3, with the eight unequal thickness layer cloak the electrical fields are smoothly excluded from the interior region and return to the original propagation direction with minimal scattering. Without the cloak, the waves are severely scattered by the bare conductor resulting in a remarkable backward reflection and evident shadow behind the conducting cylinder. Invisibility performance of the cloak with 8 layers of different thickness is quite pronounced compared to a bare conductor.

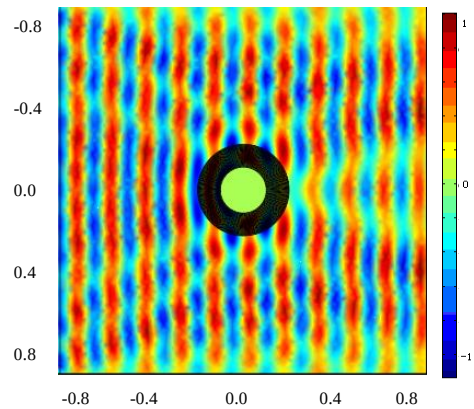
Fig.4 shows the simulation results for the electrical field around the multilayer cylindrical cloak with equal and unequal thickness (N=10). We can see that the electrical fields are smoothly excluded from the interior region with minimal scattering, the invisibility performance of two cloaks are both



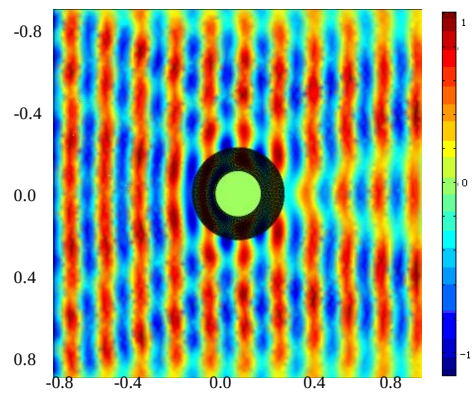
(a)

(b)

Fig3 Calculated electrical-field distribution around the conducting cylinder (a) with a cloak of 8 concentric different thickness layers, and (b) without cloak.



(a)



(b)

Fig 4 Calculated electrical-field distribution around the cloak (a) with a cloak of 10 concentric unequal thickness layers, and (b) with a cloak of 10 concentric equal thickness layers.

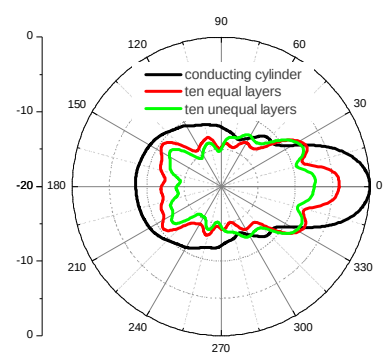


Fig 5 Far field scattering pattern (a) conducting cylinder (black line) (b) with cloak of 10 equal thickness layers (red line) (c) with cloak of 10 equal thickness layers (green line).

well. To further demonstrate the performance of the proposed unequal thickness cylindrical cloak, the far-field scattering pattern is shown in Fig 5. It can be clearly seen that with the same layers (e.g. N=10), the far-field scattering of such unequal

thickness layer structure dramatically drops compared with that of equal thickness layer structure previous publications except for some directions, the forward scattering and the backward scattering has been dramatically reduced. And these because that the parameters are well close to the discrete parameters of the anisotropy materials based on coordinate transformation.

III. Conclusions

In summary, we have successfully constructed a cylindrical cloak with structure of different thickness layer according to the slope of original ideal cylindrical cloak parameters which are spatial invariant axial material parameter. Compared to the cloak previously publications, that is, the cloak with the equal thickness layers, the cloak with the different thickness layers exhibits better invisibility performance, the far-field scattering of such structure cloak drops except for some directions, and the forward scattering has been reduced much. And these because that the parameters are well close to the discrete parameters of the anisotropy material based on coordinate transformation. Such a cloak can realize by thin layers of normal materials or composites, it can potentially be a better way for realizing a near-invisible cloak.

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