Generation of Nondiffracting Beams in Millimeter Wave Band by Diffractive Elements

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***Abstract*-A dielectric diffractive element, which can convert an incident plane wave at millimeter wavelength to be a Mathieu beam, is designed. The transverse intensity pattern of the generated Mathieu beam is plotted. The Mathieu beam has a highly localized distribution along one of the transverse directions and a sharply peaked quasi-periodic structure along the other. This nondiffracting beam can propagate a long distance without change of its transverse intensity distribution.**

I. INTRODUCTION

The nondiffracting beams have attract many researchers' interests in the past decades. Under ideal conditions, the transverse intensity distribution of the nondiffracting beams is highly concentrated on the propagation axis and the beams can propagate a long distance without suffering from the diffraction. Their potential applications in wireless power transmission, optical interconnections, laser machining, and imaging make them very relevant.

The first class of nondiffracting beams was introduced by Durnin et al. These fields were described in terms of nonsingular Bessel function, so they were named Bessel beams [1]. The zero-order Bessel beam has the maximum intensity in the beam center and is always axial symmetry. For some applications, such as in the nondestructive testing, a beam with a strip spot can accelerate the testing efficiency, so some other nondiffracting beams must be investigated.

Since the Durnin's work on Bessel beams in 1987 [1], [2], many studies about the nondiffracting beams in optical domain have been done [3]. Most of the published works consider the basic Bessel, Bessel-Gauss, and Gaussian beams. The Helmholtz equation is known to be separable in eleven coordinate systems. By solving the Helmholtz equation in elliptical cylindrical coordinate system, the Mathieu beams can be obtained. In 2000, Gutierrez-Vega et al. demonstrated the existence of the optical Mathieu beams experimentally [4], [5].

The generation of nondiffracting Bessel beams at millimeter wavelength has been realized by Y. Z. Yu and W. B. Dou [6], [7]. This paper continues to generate another nondiffracting beams - the Mathieu beams. The general properties of the Mathieu beams are discussed. A three dimensional diffractive element is proposed to generate the Mathieu beams in millimeter wave band and the transverse intensity distribution of the generated Mathieu beams is shown.

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II. THE MATHIEU BEAMS

In the free space, the scalar Helmholtz equation is

$$\left(\nabla^2 + k^2\right)E = 0 . \tag{1}$$

When expressed in the elliptical cylindrical coordinate system, this equation becomes,

$$\left[\frac{2}{p^2\left(\cosh 2\xi - \cos 2\eta\right)}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right) + \frac{\partial^2}{\partial z^2} + k^2\right]E(\xi,\eta,z) = 0 \tag{2}$$

where the elliptical cylindrical coordinate is defined by $x = h\cosh\xi\cos\eta$, $y = h\sinh\xi\sin\eta$, and $z = z$. ($\xi \in [0,\infty)$ and $\eta \in [0,2\pi)$ are the radial and angular variables respectively, and h is the semifocal distance).

Substituting the form $E = G(\xi)F(\eta)Z(z)$ into (2) and using the separation of variables, the following three equations about the three variables ξ,η,z can be obtained:

$$\left(\frac{d^2}{dz^2} + k_z^2\right)Z(z) = 0 \tag{3}$$

$$\left[\frac{d^2}{d\xi^2} - \left(h - 2q\cosh 2\xi\right)\right]G(\xi) = 0 \tag{4}$$

$$\left[\frac{d^2}{d\eta^2} + \left(h - 2q\cos 2\eta\right)\right]F(\eta) = 0 \tag{5}$$

where k_z is the wave vector component on z direction, $q = k_\tau^2 h^2/4$, and $k_\tau^2 = k^2 - k_z^2$.

Equation (3) has the solution

$$Z(z) = \exp(\mp jk_z z) \tag{6}$$

and (4) and (5) are known as the radial and angular Mathieu differential equations. Their solutions are the radial Mathieu function and angular Mathieu function, respectively.

Thus, the solution of the Helmholtz equation (2) in the elliptical cylindrical coordinate system can be expressed as,

$$E(\xi,\eta,z)=\begin{cases}\sum_{m=0}^{\infty}A_{mq}Ce_m(\xi,q)ce_m(\eta,q)\exp(\mp jk_z z) & (even)\\ \sum_{m=1}^{\infty}B_{mq}Se_m(\xi,q)se_m(\eta,q)\exp(\mp jk_z z) & (odd)\end{cases} \tag{7}$$

From (7), for any order m and parameter q, the time averaged intensity of the even mode Mathieu beams can be expressed as,

$$\begin{aligned} I(\xi,\eta,z\geq 0)&=\frac{1}{2}\left|A_{mq}Ce_m(\xi,q)ce_m(\eta,q)\exp(-jk_z z)\right|^2\\ &=\frac{1}{2}\left|A_{mq}Ce_m(\xi,q)ce_m(\eta,q)\right|^2\\ &=I(\xi,\eta,z=0)\end{aligned} \tag{8}$$

This indicates that the transverse intensity distribution of the Mathieu beam keeps the same as the beam propagates along the z axis. Thus, this beam is known as non-diffraction.

From (8), the transverse intensity patterns of the zero-order even mode Mathieu beam with different parameter q are plotted in Fig.1 (a)-(c).

It can be seen that, the transverse patterns of the Mathieu beam are highly localized in the angular direction and have a quasi-periodic structure along the radial direction. With a large parameter q, the periods in the radial direction become sharply peaked and the beam has a strip spot in the y direction.

III. The Generation of Mathieu Beams

As in Fig.2, a diffractive element is proposed to convert an incident plane wave to be the zero-order even mode Mathieu beams.

The diffractive element is a circular disc with the diameter of 40mm and average thickness of 3λ at W band. The dielectric constant of the element is 2.2. The aperture of the element is mesh with small square grids with the maximum edge length of $\lambda/3$. The thicknesses in every grid are well designed to control the phase distribution of the electromagnetic field on the back surface of the element. The radial grids control the radial phase distribution as the radial Mathieu function and the angular grids control the angular phase distribution as the angular Mathieu function.

The transverse intensity pattern of the converted zero-order Mathieu beam with the parameter q=5 in the plane with a distance of 60mm to the element is shown in Fig.3. It is seen that the aperture intensity distribution has a good agreement with the theoretical transverse distribution in Fig.1 (a).

The thickness distribution of the designed diffractive element is plotted in Fig.4. For the Mathieu beam with the parameter q=5, the oscillation in the radial direction is mild. However, when the parameter q is very large, the oscillation is very sharp, and the aperture must be meshed with smaller grids.

In optical region, the CGH (computer generated holograms) is always used to generate laser beams, which is based on the magnitude modulation. The proposed diffractive element can be seen as a phase modulation element to generate the Mathieu beams. Compared with CGH, the diffractive element has higher conversion efficiency at millimeter wavelength.

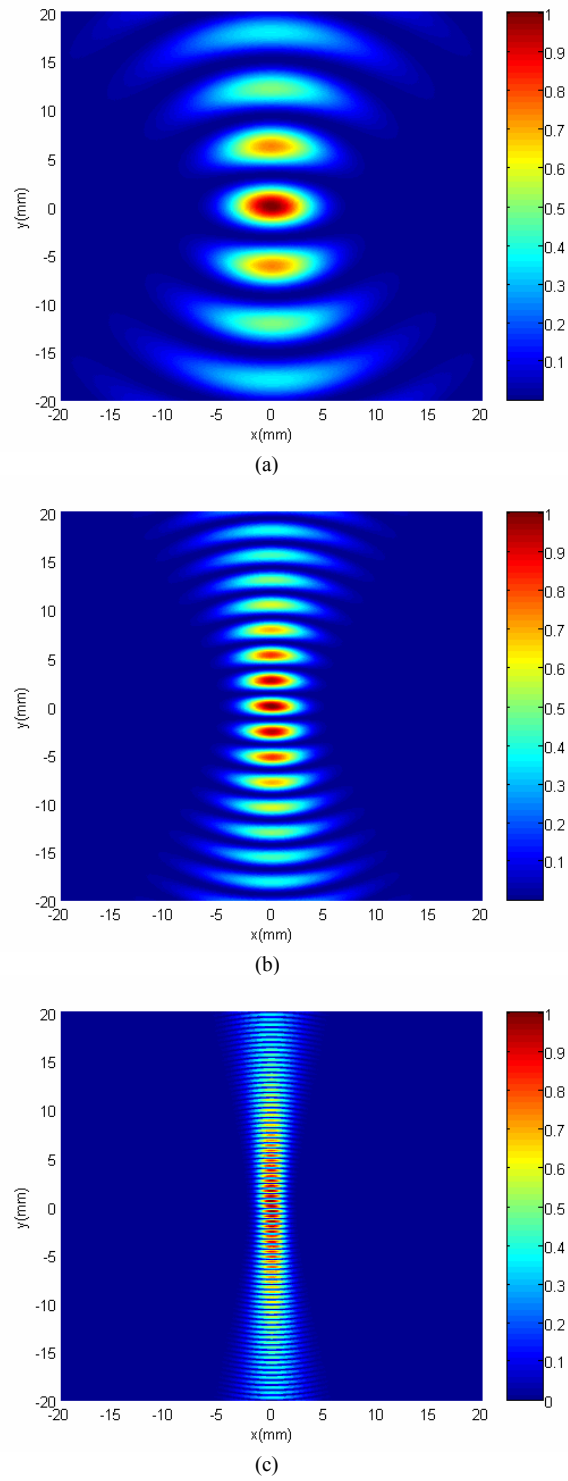


Fig. 1. The transverse intensity patterns of the zero-order Mathieu beam with different parameter q, (a) q=5, (b) q=25, (c) q=595.

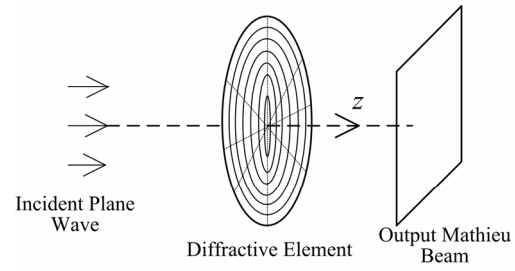
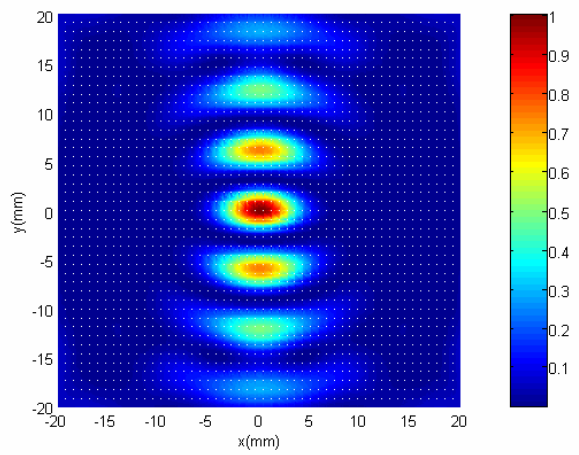
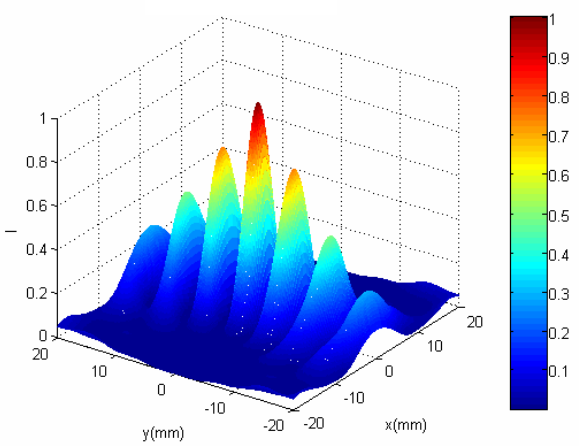


Fig. 2. The generation of the Mathieu beams by diffractive elements



(a)



(b)

Fig. 3. The generated zero-order Mathieu beam, (a) the normalized transverse intensity distribution of the generated Mathieu beams, (b) the three dimensional view of the normalized transverse intensity distribution.

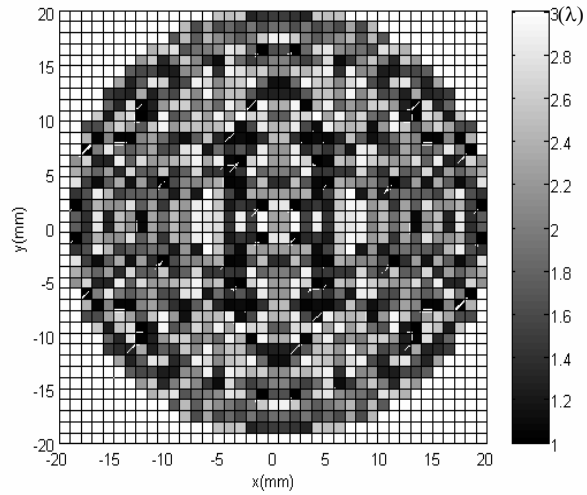


Fig. 4. The thickness distribution of the diffractive element.

IV. Conclusions

The nondiffracting Mathieu beams solution of the Helmholtz equation in the elliptical cylindrical coordinate system are deduced. This beam is highly localized in angular direction and with quasi-periodic in the radial direction. By optimizing the thickness of the diffractive element, an incident plane wave is converted to be the zero-order Mathieu beam efficiently.

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