Application of Gaussian Beam Modes and *ABCD* Matrices in the Design of Compact Array Systems

J. A. Murphy[1], M. Whale[2], A. Murk[2] and M. Renker[2].
1.National University of Ireland Maynooth, Co. Kildare Ireland.
2. Institute of Applied Physics, University of Bern, CH 3012-Bern Switzerland.

***Abstract*-In this paper we consider issues associated with the design of compact small format array systems for space satellite applications at millimeter and submillimeter wavelengths. Often such systems require folded optics with multiple mirrors which because of volume constraints may be in the near field of images of either the focal plane or the aperture plane of the satellite telescope antenna system. This implies that geometrical ray tracing will be limited in providing a good initial design concept. The behavior of the system requires including the effects of both diffraction of the individual beams of the array along with the imaging behavior of the system as a whole in mapping the region of interest. With this in mind an analytical approach based on Gaussian Beam Mode analysis and ABCD matrices is considered in this paper which can be used to search for possible solutions in the optical system design process. This approach provides an efficient means of identifying the range of allowable configurations that satisfy any mechanical constraints, as well as the mapping requirements for the scientific observations.**

I. Introduction

The modeling of compact optical systems for small format arrays in the millimeter and submillimeter bands for satellite telescope applications requires careful consideration. For imaging systems the effects of diffraction on the individual beams of the array need to be taken into account to obtain high resolution and sensitivity for observing some region of interest. At the same time within any mechanical constraints we need to optimize the mapping capability of the system as a whole in which the principle rays of the individual beams of the array can be regarded as propagating as geometrical rays. However, for a compact system some focusing components will inevitably be in the near field Fresnel region of beam waists, implying that geometrical optics alone will be inadequate in designing the system. The telescope antenna beams may also even be elliptical for some applications, for example [1], and their behavior astigmatic within the optics, thus requiring careful analysis.

In order to identify possible design configurations we consider an approach based on the application of Gaussian beam mode analysis to take account of diffraction combined with ray tracing which charts the principal axes of the individual beams. A formulism that neatly combines both approaches is based on the use of so called *ABCD* matrices, which were originally developed for describing the propagation of rays through a system but also as it turns out predict the evolution of the diffracting Gaussian beam modes. This in fact has its basis in the description of a Gaussian beam as a complex point source, as described in [2].

In order to illustrate the power of such an approach we consider the design of a system for coupling to a flat array of detectors as well also more general systems, including the case of an astigmatic configuration with elliptical input telescope beams to couple to a set of circularly symmetric detector feed beams (e.g. as in [1].) We show how it is possible using *ABCD* matrices to derive useful solutions given the mechanical constraints on the optics, so as to obtain the desired output (e.g. efficient coupling to the detector array with high resolution for the telescope in imaging applications). Such an approach is particularly useful for systems where the required beams on the sky have a significant degree of ellipticity such as for atmospheric physics or remote sensing. Given that the *ABCD* formulism assumes paraxial propagation, the approach can then be supplemented with physical optics modeling for the realistic second order behavior of the design concept.

In section II we discuss the theoretical basis for the use of *ABCD* matrices and Gaussian Beam modes in the modeling of array systems. In section III we consider the example of optimizing the coupling of a flat array to a telescope while in Section IV we investigate the optimization more general systems including astigmatic beams. Finally in Section V we draw conclusions to the work.

II. Modeling Arrays with *ABCD* Matrices & Gaussian Beam Modes.

If we assume that beams are propagating paraxially, then Gaussian beam mode analysis provides quite an accurate description of the propagation, provided any aberration and truncation effects are not significant. The Gaussian modes effectively form a basis set for any general field, and thus to a good approximation we can write the field as a sum of modes: $E(x,y) = \Sigma_m A_m E_m$, where for computational efficiency the sum contains as few terms as possible. Nevertheless, a number of higher order terms will generally be required to model a beam to reasonable accuracy. Gaussian Beam Modes have the following form in Cartesian coordinates (and are known as Hermite-Gaussian modes [3]):

$$E_{mn}(x,y,z\,;W,R,\phi_{mn}) = \frac{C_{mn}}{W} H_m\left(\frac{\sqrt{2}x}{W}\right) H_n\left(\frac{\sqrt{2}y}{W}\right) \exp\left(-j\left[kz + \frac{x^2+y^2}{2q}\right] + j\phi_{mn}\right), \quad (1)$$

where the beam width parameter W, the complex phase curvature term q and the higher order phase slippage term ϕ_{mn} depend on z, the distance along the axis of propagation. For Hermite-Gaussian modes $\phi_{mn} = (m+n+1)\phi_0$, where ϕ_0 is the phase slippage term for the fundamental mode. The complex radius of curvature q is given by $1/q = 1/R - j\lambda/\pi W^2$, the real part being the physical phase curvature and the imaginary part giving the Gaussian amplitude envelope dependency $E_{mn} \propto \exp(-(x^2+y^2)/W^2)$. C_{mn} is a normalisation factor (see for example [3]).

In order to be able to propagate a general beam we need to know how W, R and ϕ_0 change between some input and output planes of interest. We can then reconstruct the field at the output plane, where W_{OUT}, R_{OUT} are the common values for the beam width parameter and the phase curvature for the basis mode set at that plane, respectively. Clearly, critical also is how the phase slippage ϕ_{mn} term has changed between the two planes, as this determines the precise structure of the field [3].

An efficient method for determining how the beam parameters evolve is based on an application of the *ABCD* transfer matrices from paraxial ray tracing [2], [3]. According to this scheme a ray incident on an input plane at a distance of r_{IN} off-axis and travelling in a direction θ_{IN} with respect to the optical axis will emerge at the output plane with the corresponding r_{OUT} and θ_{OUT} (see Fig. 1) given by a relationship of the form:

$$\begin{pmatrix} r_{OUT} \\ \theta_{OUT} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} r_{IN} \\ \theta_{IN} \end{pmatrix}. \tag{2}$$

ABCD matrices for individual focusing components, dielectric media interfaces and inter-component propagation have simple forms. The *ABCD* matrix for a complete system can be readily computed by matrix multiplication of the cascaded individual *ABCD* matrices [3]. Note that for a general system any possible value for the coefficients of an *ABCD* matrix are in theory possible, subject to the limitation that $AD - BC = 1$.

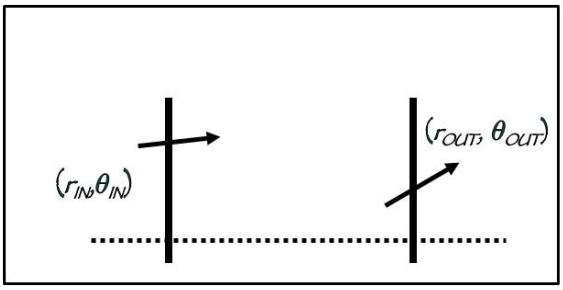

Figure 1. Definition of r and θ for the input and output planes. The *ABCD* matrix determines the transformations between the two planes.

Conveniently a Gaussian beam can also be regarded as a complex point source with a complex radius of curvature that evolves in the same way as the normal radius of curvature for an idealized point source on-axis in geometrical optics. This allows one to use the following relationship between q at the input and output planes [3]:

$$q_{OUT} = \frac{Aq_{IN} + B}{Cq_{IN} + D}, \tag{3}$$

At the same time the phase slippage evolution for the fundamental mode between the input and output planes is given by [3]: $\Delta\phi_0 = -\arg(A + B/q_{in})$, from which $\Delta\phi_{mn}$ can be derived. Thus, all the properties of a propagating mode set are readily recovered.

For an array we can model the propagation of the principle ray for the off-axis beams using ray tracing, again taking advantage of *ABCD* matrices. At the same time the precise evolution of each beam pattern with respect to its principle axis is the same for all beams and so can be derived for the on-axis beam using an on-axis mode set. This provides an efficient way computationally of describing the beam using a relatively small mode set, while tracing the principle rays of the beams is straightforward.

Alternatively, since Gaussian beam modes form a complete mode set for any paraxial field we can also model off-axis beams in an array using *any* on-axis mode set. In this case the higher order modes are critical to the description of the propagation. The precise interference between the higher order modes, controlled by the phase slippage term, determines the precise location of the main lobe of an off-axis beam, as well as also its direction of propagation (given by the phase tilt of the beam). We should choose the beam parameters for this mode set in order to be able to model any beam of the array with reasonable computational efficiency and so not just optimized for the on-axis beam. A method for determining this mode set is discussed, for example in [4] (see also [5]). The disadvantage of this technique is a relatively large mode set may be required to achieve sufficient accuracy. This ultimately limits the size of an array that can be efficiently modeled in this way. In any case the two approaches give precisely the same predictions for the evolution of the beam pattern of the array as a whole.

Using either approach we can follow the evolving beam pattern and its off-axis displacement as it propagates through an optical system. As illustrated in Fig. 2 (a) top plot, an individual beam develops sidelobe structure as it propagates and we actually obtain an image of the far field pattern on the output focal plane of the lens (where the Fourier plane of the phase centre of the feed located). In the vicinity of the Fourier plane the beams overlap and interference effects will result if the beams are mutually coherent (Fig. 2 (b) middle plot). If the power of the lens/mirror is high enough each beam will also go through a waist at some plane, where the beam intensity will be a

maximum. Furthermore, an image of the input fields at the array of waists (see for example Fig. 2 (b) middle and (c) bottom plots) will be formed at the geometrical image of the array. We next specifically consider how we can understand this behavior in terms of *ABCD* matrices and how the location of the image and Fourier planes of the array are related to the matrix coefficients.

III. Coupling to Flat Arrays using *ABCD* Matrices.

Now let us take as an example the matching of a telescope system to an array of short horns on a flat plane. In terms of optimizing the optical design an array system in particular, we need to form an image of the telescope focal plane at the detector feed waists for optimized resolution of the field being observed. At the same time an image of the array far-field patterns should coincide with the telescope aperture for good control of spill-over and edge taper. Also for optimized spill-over efficiency for the array we require all the beam trajectories to intersect at the centre of the primary mirror. With this in mind in the design process we re-image the primary with an appropriate magnification factor to some plane to which we there couple the array of detector beams.

Let us consider how we now achieve this with a single focusing component (mirror or lens) of focal length *f*, a distance *d* from the array (the input plane). The output plane is a distance *z* beyond the lens and the corresponding *ABCD* matrix is:

$$\mathbf{M} = \begin{pmatrix} 1-\dfrac{z}{f} & z+d-\dfrac{zd}{f} \\ -\dfrac{1}{f} & 1-\dfrac{d}{f} \end{pmatrix} \tag{4}$$

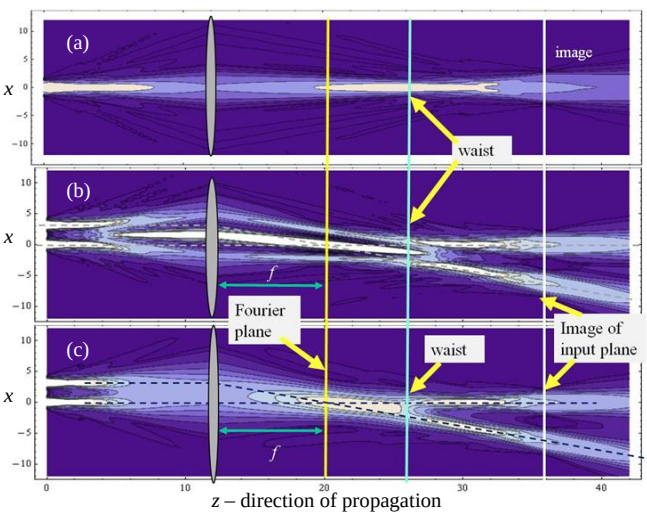

Figure 2. This figure illustrates the effects of propagation of an on-axis beam (a) and an off-axis beam (b) & (c). In the middle plot (b) the overlapping fields add coherently while in the bottom plot (c) they add incoherently. A single on-axis beam is shown in top plot (a) to emphasize the location of the waist.

Clearly, we obtain $A = 0$, when $z = f$ (the output focal plane), independent of the choice of *d*. We note that for this case the phase slippage of the fundamental satisfies: $\Delta\phi_0 = -\arg(A + B/q_{IN}) = \pm\pi/2$ *provided there is a waist at the input plane* as then $1/q_{IN}$ is pure imaginary. This phase slippage of $\Delta\phi_0 = \pm\pi/2$ also occurs between a waist and its far field (as in that case with no lens $A = 1$, and also for the far-field $B = z \to \infty$, so that $|B/q_{IN}| >> A$). Thus, at the output focal plane of the lens a Fourier transform of the feed waists (i.e. an image of the far field pattern of the horn array) is formed *no matter where the horn array is placed with respect to the lens.*

Now considering the properties of an array of beam we see that for a *flat array* producing beams propagating parallel to the central axis of the system, the focal point also corresponds to where the principle rays of the beams intersect (again because $A = 0$, so $r_{OUT} = 0$ as $\theta_{IN} = 0$, for all values of r_{IN}). Thus, this is clearly an ideal place to locate an image of the primary mirror of a telescope, if minimized spillover is required with all beams centered on the primary.

At the same time the beam widths for these images of the feed far-field patterns can be determined from the relationship: $1/q_{OUT} = (-j\pi W_{IN}^2/f^2\lambda) + (1-d/f)/f$, where we have used that $B = -1/C = f$ with $A = 0$. Thus, $W_{OUT} = \lambda f/\pi W_{IN}$ (independent of *d*), while at the same time for the phase curvature $R_{OUT} = f^2/(f-d)$. Fig. 2 (b) & (c) middle and bottom plots illustrate these points where we see the far field pattern of two beams of the array (one on-axis and the other off-axis) re-imaged onto the lens focal plane. A clear interference pattern is seen for the case of two mutually coherent sources (middle plot Fig 2 (b)). This case is also shown in more detail in Fig. 3, with cuts across the beams at various planes of interest.

One of the features of the phase slippage term is it keeps on circulating in angle in a counterclockwise sense around the unit circle as the beam propagates through the optical system, and eventually the sidelobe structure on each beam of an array actually starts to disappear as an image plane of the detector feed is approached. Such an image will be formed when $B = 0$, when $\Delta\phi_0 = -\arg(A + B/q_{in}) = -\arg(A) = \{\pi, 0\}$. It is notable that the location of any image plane is frequency independent (no dependency on λ). This is consistent with the fact that if $B = 0$, then *z* satisfies the relationship that $1/z = 1/f - 1/d$, which implies that a *geometrical optics image is also formed* for that value of *z*.

If $d > f$ then $z > f$ and the image is located beyond the focal plane (Fourier plane), whereas if $d < f$ a virtual image is formed. Also in the case of the real image $\Delta\phi_0 = \pi$ and the image is inverted (related to the symmetry of the Gaussian beam modes), whereas for the virtual image $\Delta\phi_0 = 0$ and an upright image is formed (both consistent with geometrical optics). Clearly also if *d* is very much bigger than *f*, then $B \approx 0$ when $z = f$, as expected.

Clearly from Figs. 2 and 3 we see that even though the structure of the fields at the input plane are reformed at the image plane this does not guarantee the individual beams of the array have a waist there, even if these beams have waists at the input plane. The requirement for a waist is that q_{OUT} is pure imaginary. However for the case where there is a waist at the input $1/q_{OUT} = C/A - jD\lambda/A\pi W_{IN}^2$ then $R_{OUT} = A/C$, but since C = -1/*f* in the image space of a single lens or mirror, we will therefore not obtain a waist at the image of a waist and a second focusing component will be required. This is the usual case for a Gaussian beam telescope, of course, with a waist at the input focal plane of the first lens being imaged onto a waist as the output focal plane of the second lens, but two focusing components are required.

If the input plane does not coincide with the beam waists then the beams will have non-zero curvature and in that case, we obtain a waist at the output if $R_{IN} = -D/C = f - d$. Thus, it is possible to produce waists at an image plane by arranging that $d = f - R_{IN}$. However, we note that images are virtual for a positive R_{IN}, which will be the case if the input plane is at the aperture of a short horn antenna of length *L*, (in which case $R_{IN} = L$ and $d = L - f$, the focus of the lens coincides with the apex of the horn). Nevertheless a virtual image of a horn aperture with zero phase curvature may still be a very useful input for any subsequent re-imaging optics (such as a Gaussian Beam Telescope arrangement, for example [3]).

In summary, for a propagating array of beams there are essentially 3 planes of interest (as illustrated in Figs. 2 & 3). The image plane of the array, formed at the geometrical image, the waist plane, where the individual beams go through their individual waists and the Fourier plane of the array, where the far field patterns of the feed waists of the array are formed. For a flat array the far field pattern of the feeds are formed on the focal plane of the focusing component and furthermore the beam trajectories intersect at the focal point.

IV. Optical Systems subject to Mechanical Constraints.

For a system of several focusing components there are many degrees of freedom possible for choosing the component parameters and locations though some will inevitably be limited by mechanical constraints. Various arrangements are then possible depending on the mechanical degrees of freedom. In general we must match the beam parameters *W* and *R* of the telescope at the input plane say with the feed array beams at the output plane. At the same time is it important to remember that we also need to keep control of the phase slippage term $\Delta\phi_0$ between the two planes to ensure correct imaging properties for the system (such as the telescope focal plane being imaged onto the feed horn array, for example).

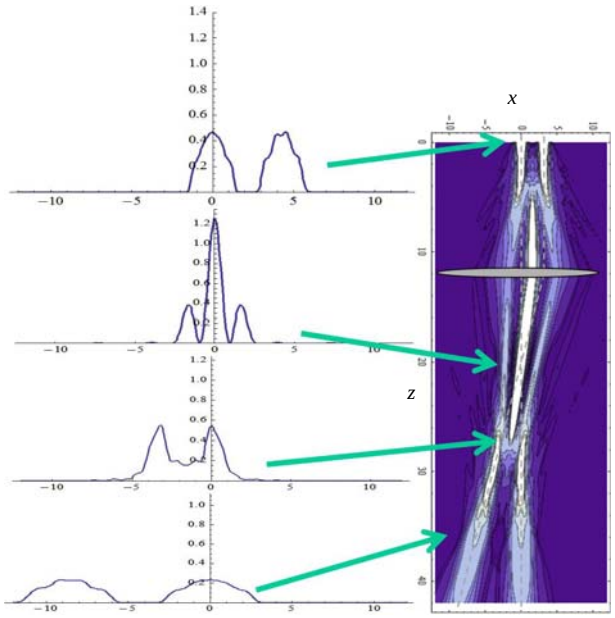

Figure 3 showing details of the on-axis and an off-axis beam of an array overlapping for a coherent example showing the beams at the input, Fourier, waist and image planes.

In general therefore we need 3 degrees of freedom. Clearly there is the possibility of obtaining these from the 3 independent values for the matrix coefficients (*A*, *B*, *C* and/or *D*) provided by some arbitrary general optical system and thus produce the correct transformations for *W*, *R* and $\Delta\phi_0$ between well defined input and output planes.

For case of *two focusing components* of focal lengths f_1 and f_2 (f_1 being the closer to the input plane), for example, which can be *separated by a variable distance z* the corresponding *ABCD* matrix gives 3 degrees of freedom:

$$\mathbf{M} = \begin{pmatrix} 1 - \dfrac{z}{f_1} & z \\ -\dfrac{1}{f_1} - \dfrac{1}{f_2} + \dfrac{z}{f_1 f_2} & 1 - \dfrac{z}{f_2} \end{pmatrix}, \qquad (5)$$

which is convenient in that the relationships between *A*, *B*, and *D* and the component parameters are easy to determine.

An adjustable output or input distance can provide extra degrees of freedoms. In any case if the optical components are not located at the input and output planes being matched, clearly we need to include in the matrix the distances to these planes yielding the following more complex *ABCD* matrix:

$$\mathbf{M} = \begin{pmatrix} 1 - \dfrac{z}{f_1} - \dfrac{d_2}{f_{com}} & z\left(1 - \dfrac{d_1}{f_1} - \dfrac{d_2}{f_2}\right) + d_1 + d_2 - \dfrac{d_1 d_2}{f_{com}} \\ -\dfrac{1}{f_{com}} & 1 - \dfrac{z}{f_2} - \dfrac{d_1}{f_{com}} \end{pmatrix}, \qquad (6)$$

where $1/f_{com} = 1/f_1 + 1/f_2 - z/f_1 f_2$ and d_1 is the distance from the input plane to the first mirror ($f = f_1$) and d_2 is the distance from the second mirror ($f = f_2$) to the output plane

Another situation which may occur is where the locations of the focusing components are mechanically fixed and therefore 3 components are required to match input and output planes for general fields. Thus, if the inter focusing distances are z_1 between focusing components 1 & 2 and z_2 between lens 2 & 3 (and f_1 is again the closest to the input plane), we obtain an *ABCD* matrix of the form:

$$\mathbf{M} = \begin{pmatrix} 1 - \frac{z_1 + z_2}{f_1} - \frac{z_2}{f_2} + \frac{z_1 z_2}{f_1 f_2} & z_1 + z_2 - \frac{z_1 z_2}{f_2} \\ -\frac{1}{f_1} - \frac{1}{f_2} - \frac{1}{f_3} + \frac{z_1 + z_2}{f_3}\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \frac{z_1 z_2}{f_1 f_2 f_3} & 1 - \frac{z_1}{f_2} - \frac{z_1 + z_2}{f_3} + \frac{z_1 z_2}{f_2 f_3} \end{pmatrix}, \quad (7)$$

This has the convenient property that for a system with constrained component locations, and so predetermined distances z_1 and z_2, we can calculate the respective required powers ($P = 1/f$) of the corresponding focusing components as follows:

$$\begin{pmatrix} 1 - (\alpha + \beta)P_1 - \beta P_2 + \alpha\beta P_1 P_2 & \alpha + \beta - \alpha\beta P_2 \\ -P_1 - P_2 - P_3 + (\alpha + \beta)P_3(P_1 + P_2) + \alpha\beta P_1 P_2 P_3 & 1 - \alpha P_2 - (\alpha + \beta)P_3 + \alpha\beta P_2 P_3 \end{pmatrix}, \quad (8)$$

where $\alpha = z_1$ and $\beta = z_2$ are predetermined (in convenient units of distance in these formulas). Clearly, for the required *ABCD* matrix $P_2 = (\alpha + \beta - B)/\alpha\beta = (z_1 + z_2 - B)/z_1 z_2$, from which P_1 and P_3 can be computed and thus the required system determined.

We can also consider more complex arrangements in which we are required to couple a set of astigmatic beams such as produced by a telescope with an elliptical aperture to an array of circular beams such as the case of a corrugated horn array, for example. Therefore between the two planes we have the requirement to match the W of the circular horn array beam waists to the two widths W_x and W_y of the elliptical telescope beams, where we have aligned the local x and y axes with the major and minor axis of the ellipse defined by these beams. As well as matching the beam widths we are also required to match the phase radii of curvatures and the phase slippage parameters ($\Delta\phi_{0x}$, $\Delta\phi_{0y}$) for the elliptical beams between the input plane and the output planes (phase slippage is critical for imaging with total phase slippage for a skew mode being $\Delta\phi_{mn} = (m+½)\Delta\phi_{0x} + (n+½)\Delta\phi_{0y}$). Within the system we assume the propagation of the beams can be described in terms of a skew Gaussian beam mode set defined in terms of independent beam parameters (W_x, R_x, ϕ_x) and (W_y, R_y, ϕ_y). For each orientation we require 3 degrees of freedom, so that the minimum number of optical components required to achieve the match, if their locations are fixed by mechanical constraints is 3, where each component has different focal lengths f_x and f_y in the two principle orientation directions of the skew beams. Clearly, if there are no mechanical constraints or limitations present it may be possible to match the beams with less than 3 components, but the required distances to the object and image for a match may be mechanically awkward.

This approach was taken for the example shown in Fig. 4, (based on a system defined in [1]), in which we wish to form an image of the telescope aperture (for a Cassegrain arrangement) at some convenient plane in the optical system to which we couple the array of feed beams. We assume that the system is subject to mechanical constraints with regard to the positioning of the focusing components. On the array side mirror M_A of focal length f_A produces a Fourier pattern at its output focal plane, which is constrained to be a certain distance d_2 from the mirror M_3 (with focal lengths f_{3x}, f_{3y}). The beam width parameter at the aperture image plane is W_F (where the beams are non-astigmatic) is given by $W_F W_A = \lambda f_A / \pi$, where W_A is the beam size at the waist at the detectors, while $R_F = f_A^2/(f_A - d_A)$ for the array beams there also. Note that the telescope itself produces an array of astigmatic elliptical sky beams at its Cassegrain focus with primary (M_P) and secondary mirrors (M_S). However, before the focus is formed a relay of 3 mirrors (M_1, M_2 &M_3) is introduced in which the positions of the mirrors are fixed, but their pairs of focal lengths f_x and f_y are adjustable. We can then determine the pair of *ABCD* matrices $\mathbf{M_x}$ and $\mathbf{M_y}$, that brings us from the telescope primary to the aperture image plane. At the same time we know we require a magnified circularly symmetric image of the array far fields at the primary mirror, which defines a full *ABCD* matrix for the system as a whole from the telescope to the array.

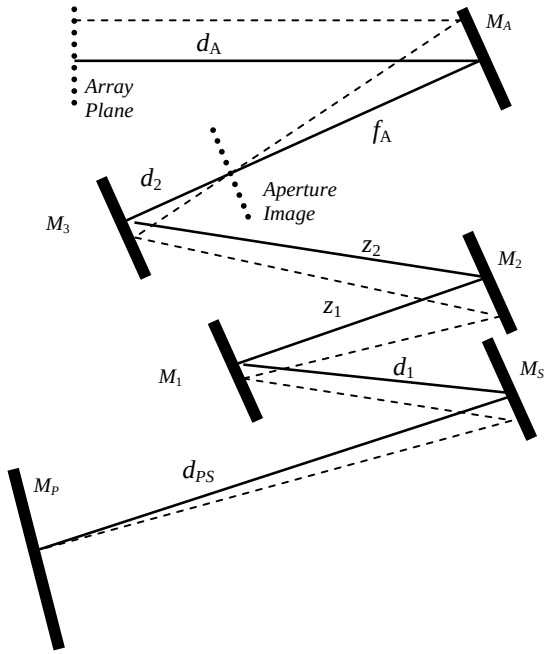

Figure 4: Schematic of the astigmatic optical system (based on [1]).

TABLE I
BEAM PARAMETERS AT, AND FOCAL LENGTHS OF VARIOUS MIRRORS

Mirror	W_x (mm)	W_y (mm)	f_x (mm)	f_y (mm)	$\Delta\phi_{0,x}$ (rad)	$\Delta\phi_{0,y}$ (rad)
M_P	471	235	1105.9		0	
M_S	58.2	29.1	-151.5		0.01	0.08
M_1	37.2	19.4	-2150.1	-17×10^3	0.08	0.29
M_2	23.5	13.0	681.0	542.8	0.26	0.99
M_3	14.1		255.0	493.1	2.84	
Image Plane	14.7		$d_2 = 213.8$		$3.14 = \pi$	
Array Plane	10.2		$f_A = 526.4$		$-1.57 = \pi/2$	

Table 1 lists the beam parameters at the various mirrors and at the image plane of the telescope aperture. As can be seen the required phase slippage is the same for both beam orientations and the beams are symmetric after M_3, as required. Note that beam waists are formed at M_3, while the image of the telescope aperture is at a distance d_2 away. Thus, a solution is arrived at through the *ABCD* matrix analysis as described above. This solution is illustrated in Fig. 5 (a) & (b). In Fig.6 we show typical far field plots for beams on the sky for a realistic typical system when a full physical optics analysis is included (based on [1]).

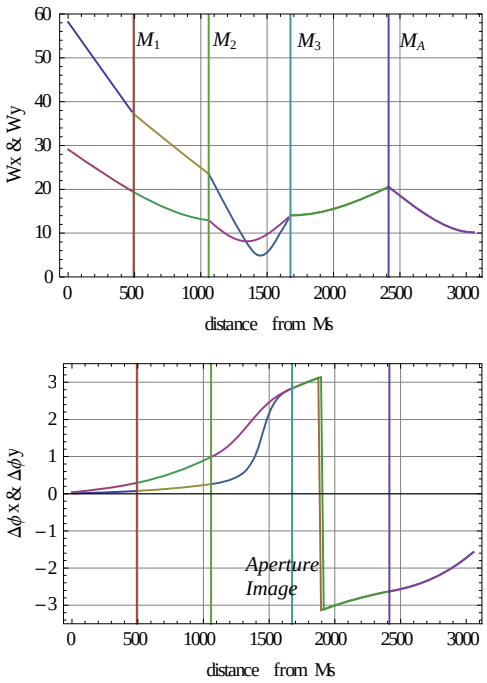

Figure 5: Evolution of beam parameters in an astigmatic optical system (see Table I) with distance from M_S. Positions of the mirrors are indicated. At image plane $\Delta\phi_0$ jumps from $+\pi$ to $-\pi$. At array plane $\Delta\phi_0 = -\pi/2$ (equivalent to $3\pi/2$).

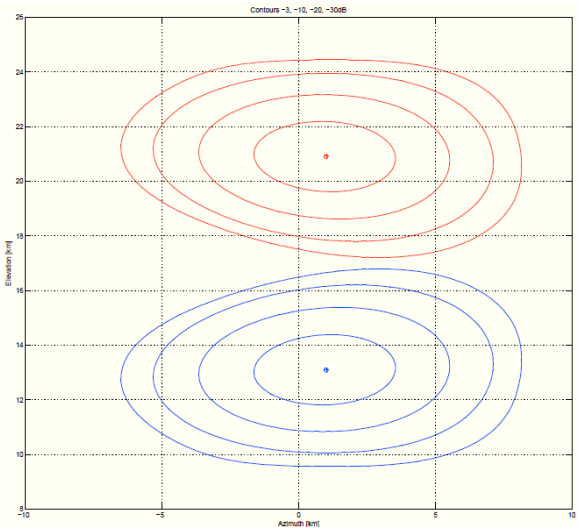

Figure 6: Typical far field beam patterns for 2 example beams in an array based on a physical optics analysis for the beams on the sky. An elliptical telescope feeds an astigmatic quasi-optical system coupled to a feed horn array (based on a system described in [1]). Contour levels at -3dB, -10dB, -20dB and -30dB.

V. CONCLUSIONS.

In this paper we have reported on an approach for optimizing the design of small compact arrays using *ABCD* matrices. We have shown that for mechanical constraints the requirements of the optical system can be expressed conveniently in terms of *ABCD* matrices which provide an analytical framework for optimizing design concepts. The precise optical implication in terms of sidelobes, aberrations and cross polarization levels are then extracted using accurate techniques such as physical optics.

ACKNOWLEDGMENT

This work was supported by the Swiss National Science Foundation under Grant 200021-130517 and the ESA PRODEX project C4000100544.

REFERENCES

[1] M. Whale, M. Renker, A. Murk, O. Janson, U. Frisk, "Optical Design of the Stratosphere-Troposphere Exchange and Climate Monitor Radiometer (STEAMR) Instrument", 5th European Conference on Antennas and Propagation (EUCAP 2011), Rome, Italy, April 2011.

[2] A. E. Siegman, *Lasers*, University Science Books. 1986

[3] P.F. Goldsmith, *Quasioptical Systems: Gaussian Beam Quasioptical Propagation and Applications*, IEEE Press: New York. 1998.

[4] J.A. Murphy and Withington S, "Gaussian beam mode analysis of multi-beam quasi-optical systems," SPIE Conference on Advanced Technology MMW, Radio, and Terahertz Telescopes, MAR 26-28, 1998, pp. 97-104.

[5] M.L. Gradziel, C. O'Sullivan, J.A. Murphy JA, et al., "Modelling of the optical performance of millimetre-wave instruments in MODAL - art. no. 64720D" SPIE Conference on Terahertz and Gigahertz Electronics and Photonics, JAN 21-22, 2007.